

T-duality in Ramond-Ramond backgrounds

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February 2, 2008

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Abstract

Using the pure spinor formalism on the world-sheet, we derive the T-duality rules for all target space couplings in an efficient manner. The world-sheet path integral derivation is a proof of the equivalence of the T-dual Ramond-Ramond backgrounds which is valid non-perturbatively in the string length over the curvature radius and to all orders in perturbation theory in the string coupling.

1 Introduction

Target space duality is a symmetry of string theory that maps a string theory in a background to a dual string theory in a dual background. For reviews see [1][2]. The map between T-dual backgrounds was derived in a world-sheet path integral formalism for Neveu-Schwarz Neveu-Schwarz fields in [3]. For Ramond-Ramond backgrounds in type II, various derivations have appeared in the literature: the authors of [4] used the equivalence of the type II supergravity actions after reduction to nine dimensions. In [5] arguments were given for the transformation of the spacetime supersymmetry parameters, and then the transformations of the gravitini and of the Ramond-Ramond fields were inferred by demanding compatibility between T-duality and supersymmetry. A world-sheet derivation was obtained in the Green-Schwarz formalism in [6] up to quadratic order in the superspace coordinate, and later extended to all orders in [7][8].

In the present letter, we give a novel world-sheet derivation of T-duality based on the pure spinor formalism [9]. Our motivation for revisiting this problem is twofold: first, the duality rules are derived in a simpler and more streamlined way than with other methods. Secondly, since the pure spinor formalism gives a satisfactory conformal field theory description of the string in generic backgrounds, we are able to promote the duality to the path integral level, thus providing for a derivation of T-duality which is valid non-perturbatively in the string length over the curvature radius and to all orders in the string coupling. The duality is valid in the presence of Ramond-Ramond and fermionic backgrounds.

2 Derivation of the classical T-duality rules

We will derive the T-duality rules from the world-sheet pure spinor formalism [9]. (See [10] for a review.) The derivation will have the advantage of simplicity, and the formalism is suitable for the full quantum theory since in the pure spinor formalism the theory can be quantized without obstruction.

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The pure spinor world-sheet action is :

$$\begin{aligned}
S = & \frac{1}{2\pi\alpha'} \int d^2z \left[\frac{1}{2} (G_{MN}(Z) + B_{MN}(Z)) \partial Z^M \bar{\partial} Z^N + P^{\alpha\hat{\beta}}(Z) d_\alpha \hat{d}_{\hat{\beta}} \right. \\
& + E_M^\alpha(Z) d_\alpha \bar{\partial} Z^M + E_M^{\hat{\alpha}}(Z) \hat{d}_{\hat{\alpha}} \partial Z^M + \Omega_{M\alpha}{}^\beta(Z) \lambda^\alpha w_\beta \bar{\partial} Z^M + \hat{\Omega}_{M\hat{\alpha}}{}^{\hat{\beta}}(Z) \hat{\lambda}^{\hat{\alpha}} \hat{w}_{\hat{\beta}} \partial Z^M \\
& + C_\alpha^{\beta\hat{\gamma}}(Z) \lambda^\alpha w_\beta \hat{d}_{\hat{\gamma}} + \hat{C}_{\hat{\alpha}}^{\hat{\beta}\gamma}(Z) \hat{\lambda}^{\hat{\alpha}} \hat{w}_{\hat{\beta}} d_\gamma + S_{\alpha\hat{\gamma}}^{\beta\hat{\delta}}(Z) \lambda^\alpha w_\beta \hat{\lambda}^{\hat{\gamma}} \hat{w}_{\hat{\delta}} \Big] \\
& + \frac{1}{4\pi} \int d^2z (\Phi(Z) \mathcal{R}^{(2)}) + S_\lambda + \hat{S}_{\hat{\lambda}}
\end{aligned} \tag{1}$$

where the coordinates $Z^M = (x^\mu, \theta^\alpha, \hat{\theta}^{\hat{\alpha}})$ are coordinates on an $\mathbb{R}^{10|32}$ superspace. The variables d and \hat{d} as well as w and \hat{w} are independent spinorial variables while the variables λ and $\hat{\lambda}$ are pure spinors, which means that they are Weyl spinors satisfying the constraint

$$\lambda^\alpha \gamma_{\alpha\beta}^a \lambda^\beta = 0. \tag{2}$$

The action for the pure spinors, $S_\lambda + \hat{S}_{\hat{\lambda}}$, is formally a free field action. Because of the constraints a proper treatment requires care. That will not be important for our purposes.

The action (1) describes both the type IIB and IIA string. The only difference is whether the hatted and un-hatted spinor indices have the same or the opposite chirality. All the couplings are superfields, which means they are generic functions of all the superspace coordinates. For the reader's convenience, we recall the meaning of the various superfields : G_{MN} is the metric, B_{MN} is the B-field, $P^{\alpha\hat{\beta}}$ are the RR field strengths, E_M^α is the spinorial part of the vielbein, $\Omega_{M\alpha}{}^\beta$ is the spin connection, $C_\alpha^{\beta\hat{\gamma}}$ contains the field strength of the dilatino, $S_{\alpha\hat{\gamma}}^{\beta\hat{\delta}}$ contains the Riemann curvature, and Φ is the dilaton. The world-sheet curvature is denoted $\mathcal{R}^{(2)}$, and it couples to the dilaton via the Fradkin-Tseytlin term in the last line. Superdiffeomorphisms and local Lorentz transformations allow to make particular gauge choices where the physical content of the fields is more manifest. For brevity's sake, we refer again to [10].

As mentioned in [10] and explained in detail in [11], the action enjoys a BRST symmetry generated by the charge

$$Q = \int (\lambda^\alpha d_\alpha + \hat{\lambda}^{\hat{\alpha}} \hat{d}_{\hat{\alpha}})$$

when the background fields satisfy a set of constraints, which are known to put all the fields on-shell.

For our purposes it is important to notice that the action (1) is the most general one that respects the local symmetries. Since, as it turns out, the BRST operator will not be affected by the T-duality, it follows immediately that the T-duality transformed fields will again solve the equations of motion. This had to be imposed as a requirement in order to find the form of the transformations in the Green-Schwarz formalism [7], but in the pure spinor formalism it is automatically true.

The difference with the Green-Schwarz string in this respect is due to the fact that the action (1) explicitly contains all the fields that appear in the superspace description of the gravity multiplet. The Green-Schwarz action instead depends explicitly only on the bosonic part of the supervielbein, so that additional input is required in order to find the transformation rules for the other fields. In the pure spinor formalism, when we perform the T-duality transformation on the action, we are bound to find an action of the same form as (1), and we can directly read off the transformation rules for all superfields.

For now we concentrate on the first three lines of the action (1) – we come back to the last line later on. We suppose that a Killing vector exists in space-time, and we choose local coordinates such that all space-time superfields do not depend on the coordinate x^1 . The tangent vector $\partial/\partial x^1$ is proportional to the Killing vector in a given patch. The action then has a global shift symmetry (in x^1) that we gauge by introducing a gauge field one-form A on the world-sheet.

We moreover add a term to the action that corresponds to a Lagrange multiplier y multiplying the field strength $F = dA$:

$$S_{gauged}[\partial x_1, A, y] = S[\partial x^1 - A] + \frac{1}{2\pi\alpha'} \int d^2z y (\partial \bar{A} - \bar{\partial} A) \quad (3)$$

The model we obtain in this way is locally and classically equivalent to the original model, since the equation of motion for the Lagrange multiplier forces the gauge field to be pure gauge [3]. The original dynamics survives unaltered.

On the other hand, we can gauge x^1 to zero, and we obtain an action that is quadratic in the gauge field A . It is possible to integrate out the gauge field classically, but the Gaussian integral to be performed requires a regularization, that we discuss in the following section. The naive integration results in the dual action:

$$\begin{aligned} S = & \frac{1}{2\pi\alpha'} \int d^2z \left\{ \right. \\ & \frac{1}{2} \left[\left(\frac{4}{G_{11}} \right) (\partial y \bar{\partial} y) + \left(-2 \frac{G_{1M} + B_{1M}}{G_{11}} \right) (\partial y \bar{\partial} Z^M) + \left(2 \frac{G_{M1} + B_{M1}}{G_{11}} \right) (\bar{\partial} y \partial Z^M) \right. \\ & + \left(G_{MN} + B_{MN} - (-)^{MN} \frac{(G_{M1} + B_{M1})(G_{1N} + B_{1N})}{G_{11}} \right) (\partial Z^M \bar{\partial} Z^N) \left. \right] \\ & + \left(P^{\alpha\hat{\beta}} + \frac{2E_1^\alpha E_1^{\hat{\beta}}}{G_{11}} \right) d_\alpha \hat{d}_{\hat{\beta}} \\ & + \frac{2E_1^\alpha}{G_{11}} d_\alpha \bar{\partial} y + \left(E_M^\alpha - \frac{(G_{1M} + B_{1M})E_1^\alpha}{G_{11}} \right) d_\alpha \bar{\partial} Z^M \\ & - \frac{2E_1^{\hat{\beta}}}{G_{11}} \partial y \hat{d}_{\hat{\beta}} + \left(E_M^{\hat{\beta}} - \frac{(G_{M1} + B_{M1})E_1^{\hat{\beta}}}{G_{11}} \right) \hat{d}_{\hat{\beta}} \partial Z^M \\ & + \frac{2\Omega_{1\alpha}{}^\beta}{G_{11}} \lambda^\alpha w_\beta \bar{\partial} y + \left(-\frac{G_{1M} + B_{1M}}{G_{11}} \Omega_{1\alpha}{}^\beta + \Omega_{M\alpha}{}^\beta \right) \lambda^\alpha w_\beta \bar{\partial} Z^M \\ & - \frac{2\hat{\Omega}_{1\hat{\alpha}}{}^{\hat{\beta}}}{G_{11}} \hat{\lambda}^{\hat{\alpha}} \hat{w}_{\hat{\beta}} \partial y + \left(-\frac{G_{M1} + B_{M1}}{G_{11}} \hat{\Omega}_{1\hat{\alpha}}{}^{\hat{\beta}} + \hat{\Omega}_{M\hat{\alpha}}{}^{\hat{\beta}} \right) \hat{\lambda}^{\hat{\alpha}} \hat{w}_{\hat{\beta}} \partial Z^M \\ & + \left(C_\alpha^{\beta\hat{\gamma}} - \frac{2}{G_{11}} \Omega_{1\alpha}{}^\beta E_1^{\hat{\gamma}} \right) \lambda^\alpha w_\beta \hat{d}_{\hat{\gamma}} + \left(\hat{C}_{\hat{\alpha}}^{\hat{\beta}\gamma} - \frac{2}{G_{11}} \hat{\Omega}_{1\hat{\alpha}}{}^{\hat{\beta}} E_1^\gamma \right) \hat{\lambda}^{\hat{\alpha}} \hat{w}_{\hat{\beta}} d_\gamma \\ & + \left(S_{\alpha\hat{\alpha}}^{\beta\hat{\beta}} - \frac{2}{G_{11}} \Omega_{1\alpha}{}^\beta \hat{\Omega}_{1\hat{\alpha}}{}^{\hat{\beta}} \right) \lambda^\alpha w_\beta \hat{\lambda}^{\hat{\alpha}} \hat{w}_{\hat{\beta}} \left. \right\}. \end{aligned} \quad (4)$$

In the action (4) the sum over the M index is now over all variables except x^1 . The sign $(-)^{MN}$ is -1 when M and N are fermionic indices and $+1$ otherwise. It should be clear that the first two lines give the classical T-duality rules for the NSNS sector background fields. To proceed, we take a closer look at the fourth and fifth lines that code the transformation properties of the fermionic vielbeins. Given the matrices Q and \hat{Q} :

$$\begin{aligned} Q_M{}^N &= \begin{pmatrix} \frac{2}{G_{11}} & 0_{1 \times 9|32} \\ -\frac{1}{G_{11}}(G_{1M} + B_{1M}) & 1_{9|32 \times 9|32} \end{pmatrix} \\ \hat{Q}_M{}^N &= \begin{pmatrix} -\frac{2}{G_{11}} & 0_{1 \times 9|32} \\ -\frac{1}{G_{11}}(G_{M1} + B_{M1}) & 1_{9|32 \times 9|32} \end{pmatrix} \end{aligned} \quad (5)$$

we see that the supervielbeins transform as $E_M'^\alpha = Q_M{}^N E_N^\alpha$ and $\hat{E}_M'^{\hat{\alpha}} = \hat{Q}_M{}^N \hat{E}_N^{\hat{\alpha}}$. From the action we cannot directly infer the transformation rule for the bosonic vielbein, since it only

appears via the metric $G_{MN} = E_M^a E_N^b \eta_{ab}$. Either Q or \hat{Q} acting on E_M^a gives a rule compatible with the transformation of the metric. So there are two candidates for a T-dual vielbein. We note (similarly as in [5]) that the two possibilities are related by the transformation $Q\hat{Q}^{-1}$ which is a Lorentz transformation of determinant -1. In fact it is a parity transformation in the direction of the T-duality. In order to be consistent, we have to act with a parity transformation in one spinor sector. We can choose it to be in the hatted sector, the other choice being entirely equivalent. The chiral change in parity takes us from a type IIA/B background to a type IIB/A background. We can then put the action in the original form by redefining the right-moving fermions as follows:

$$\hat{\psi}' = \Gamma \hat{\psi} \quad (6)$$

where $\hat{\psi} = \hat{\lambda}, \hat{\theta}, \hat{w}, \hat{d}$ is any of the spinorial fields, and $\Gamma = \gamma_1$. It can be checked that after the redefinition of the fermionic variables the action is indeed of the original form, but the background superfields with hatted spinor indices have to be redefined. Moreover, since $\Gamma^2 = 1$, the T-duality rules leave the BRST charge invariant, as anticipated. We can now give the transformation rules for all the superfields. Combining the metric and B-field in the tensor $L_{MN} = G_{MN} + B_{MN}$, we find

$$\begin{aligned} G'_{11} &= \frac{4}{G_{11}} \\ L'_{1M} &= -2 \frac{L_{1M}}{G_{11}} \\ L'_{M1} &= 2 \frac{L_{1M}}{G_{11}} \\ L'_{MN} &= L_{MN} - (-)^{MN} \frac{L_{M1} L_{1N}}{G_{11}} \\ P'^{\alpha\hat{\beta}'} &= \left(P^{\alpha\hat{\beta}} + 2 \frac{E_1^\alpha E_1^{\hat{\beta}}}{G_{11}} \right) \Gamma_{\hat{\beta}}^{\hat{\beta}'} \\ E_M'^\alpha &= Q_M^N E_N^\alpha \\ \hat{E}_M'^{\hat{\alpha}'} &= \hat{Q}_M^N \hat{E}_N^{\hat{\alpha}} \Gamma_{\hat{\alpha}}^{\hat{\alpha}'} \\ \Omega_{M\alpha}'^\beta &= Q_M^N \Omega_{N\alpha}^\beta \\ \hat{\Omega}_{M\hat{\alpha}'}'^{\hat{\beta}'} &= \hat{Q}_M^N \hat{\Omega}_{N\hat{\alpha}}^{\hat{\beta}} \Gamma_{\hat{\alpha}}^{\hat{\alpha}'} \Gamma_{\hat{\beta}}^{\hat{\beta}'} \\ C_\alpha'^{\beta\hat{\gamma}'} &= \left(C_\alpha^{\beta\hat{\gamma}} - \frac{2}{G_{11}} \Omega_{1\alpha}^\beta \hat{E}_1^{\hat{\gamma}} \right) \Gamma_{\hat{\gamma}}^{\hat{\gamma}'} \\ \hat{C}_{\hat{\alpha}'}'^{\hat{\beta}'\gamma} &= \left(\hat{C}_{\hat{\alpha}}^{\hat{\beta}\gamma} - \frac{2}{G_{11}} \hat{\Omega}_{1\hat{\alpha}}^{\hat{\beta}} E_1^\gamma \right) \Gamma_{\hat{\alpha}}^{\hat{\alpha}'} \Gamma_{\hat{\beta}}^{\hat{\beta}'} \\ S_{\alpha\hat{\gamma}'}'^{\beta\hat{\delta}'} &= \left(S_{\alpha\hat{\alpha}}^{\beta\hat{\beta}} - \Omega_{1\alpha}^\beta \hat{\Omega}_{1\hat{\alpha}}^{\hat{\beta}} \right) \Gamma_{\hat{\gamma}'}^{\hat{\alpha}'} \Gamma_{\hat{\beta}}^{\hat{\delta}'} \end{aligned} \quad (7)$$

These transformations contain all fermionic corrections to the T-duality. Our derivation of the T-duality rules is considerably more concise than the derivations in the literature. It can be checked that when the background is on-shell, *i.e.* it satisfies the torsion constraints, the dual background is also on-shell. In this case the transformation rules are a bit simpler, because the torsion constraint

$$T_{\alpha\alpha}^\beta = 0 = \hat{T}_{\alpha\hat{\alpha}}^{\hat{\beta}}$$

together with the condition that the fields do not depend on x^1 , implies that $\Omega_{1\alpha}^\beta = 0 = \hat{\Omega}_{1\hat{\alpha}}^{\hat{\beta}}$.

The fact that an on-shell background is transformed into another on-shell background can also be argued purely in world-sheet terms. We need to show that the BRST symmetry of

the original action carries over to the dual action. Since the gauge field A and the Lagrange multiplier y are BRST invariant, and we assume that the original action is invariant as well, the total gauged action is invariant. To go to the dual theory we integrate out x_1 , which is not closed under BRST so naively we seem to break the symmetry. To show that this is not the case, we have to perform a field redefinition before integrating out; we shift the gauge field as follows:

$$\begin{aligned} A &\rightarrow A + \frac{1}{G_{11}} E_1^\alpha d_\alpha \partial x^1 \\ \bar{A} &\rightarrow \bar{A} + \frac{1}{G_{11}} E_1^{\hat{\alpha}} \hat{d}_{\hat{\alpha}} \bar{\partial} x^1 \end{aligned}$$

The effect of this shift in the action (3) is to cancel the couplings between d and ∂x^1 and replace it with a coupling between d and ∂y . As a result, the symplectic structure on the space of fields is modified, and it can be shown that the BRST charge (formally given by the same expression) now leaves x^1 invariant and acts instead on y . It is then safe to integrate out the isometry coordinate x^1 and the gauge potential A , and we arrive again at the dual action (4), having preserved the BRST invariance at each step.

In order to compare our results with those of [7] it is important to keep in mind that we are working in a different superspace. More precisely, Berkovits's formalism naturally gives the formulation of supergravity in Weyl superspace [12], since the spin connection takes values in $spin(1,9) \times \mathbb{R}$ (corresponding to the 0 and 2-form part of $\Omega_{M\alpha}^\beta$ viewed as a matrix in the Clifford algebra; the 4-form part has to vanish for the action to be gauge-invariant). While it is possible to reduce the structure group to the Lorentz part only, this has some consequences on the structure of the torsion constraints. In particular, in ordinary superspace one of the constraints reads

$$T_{\alpha\beta}^\gamma = \left(\delta_{(\alpha}^\gamma \delta_{\beta)}^\rho + (\gamma_a)_{\alpha\beta} (\gamma^a)^{\gamma\rho} \right) \Lambda_\rho$$

where Λ is the dilatino. In Weyl superspace it is possible to set $T_{\alpha\beta}^\gamma = 0$, but the dilatino is then absorbed in the spin connection (see [13] for a thorough discussion).

3 Regularization and quantum equivalence

We have been careful in choosing a world-sheet formalism that can be regularized [14][15] and quantized [9]. In particular, the only quantum calculation we need to perform is the Gaussian integration over the gauge field and the coordinates. It is rigorously performed in [14][15], and we summarize those discussions.

We introduce a generalized Hodge decomposition of the gauge field on the world-sheet $A_a = \partial_a \alpha + \epsilon_a{}^b \partial_b \beta / G_{11}$. The Gaussian integrations that one performs to obtain either the original or the dual model are over the variables $x^1 - \alpha, y$ and β . The Gaussian integration can be regularized efficiently by introducing either dimensional regularization as in [16] or a Pauli-Villars regulator field for each integration variable [15], with a kinetic term determined by the quadratic terms in the fields $x^1 - \alpha, y, \beta$.

The crucial observation that we make now is that in the pure spinor world-sheet action (for a generic background), the quadratic terms are identical to the ones in the Neveu-Schwarz Ramond formalism for a purely NS-NS background up to the fact that the coefficients are superfields in our context, and does not affect the calculation. We therefore conclude that the action for the regulator fields is identical to the regulated action in the Neveu-Schwarz Ramond formalism. Therefore, as in [15], in either regularization scheme, the Gaussian integration provides us with an equality between regularized path integrals for the dual world-sheet quantum field theories. Thus the equivalence of non-linear sigma models is valid to all orders in world-sheet perturbation theory (in the string length over the curvature radius squared), and even non-perturbatively. To establish this it is crucial to demonstrate that the non-local contributions to the regulated

actions on either side match, as demonstrated in [14]. Precisely as for purely NSNS backgrounds it can be shown that conformal models on one side of the duality are mapped into conformal dual models provided one shifts the dilaton, which takes into account the conformal anomaly. The independence of the regularization procedure on the Ramond-Ramond backgrounds is responsible for the fact that the dilaton shifts with an amount that depends on the Neveu-Schwarz Neveu-Schwarz (superfield) background only. The transformation rule is then

$$\Phi' = \Phi - \frac{1}{2} \ln G_{11} .$$

That provides the T-duality rule for the dilaton in the final line in the action (1). The pure spinor action is unaltered.

An important point we want to make is that we can do the full regularization of all fields in a dimensional regularization scheme. That scheme does not break BRST invariance of the action on both sides of the T-duality. Thus, on-shell backgrounds are mapped onto on-shell backgrounds, at the quantum level, in the dimensional regularization scheme.

To argue for T-duality to all order in the string coupling g_s , we reason as follows. Given the fact that on any given world-sheet (with given topology and modular parameters) we can demonstrate non-perturbative equivalence of the world-sheet models, it suffices to observe that the modular integrals will be identical for type IIA and type IIB string theories. We thus provide a proof of T-duality that is perturbative in the string coupling, and non-perturbative in the string length over the curvature radius, and this in any background including those with Ramond-Ramond fields.

4 Conclusions

The world-sheet pure spinor formalism enables us to efficiently derive the T-duality rules for string backgrounds, as well as to generalize the proof of equivalence of backgrounds non-perturbatively on the world-sheet and perturbatively in target space to any non-trivial background of string theory. Moreover, the duality holds for backgrounds that are on-shell or off-shell. It is interesting to further study global aspects of the duality in backgrounds with Ramond-Ramond fluxes. T-duality on superspaces using the world-sheet spinor formalism also deserves investigation. It should be possible using our techniques to analyze the important case of the AdS_5 background, that has been the subject of recent investigations (see e.g. [17][18]).

Acknowledgments We thank Costas Bachas and Dimitri Tsimpis for helpful discussions. Our work was supported in part by the EU under the contract MRTN-CT-2004-005104.

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